New Approach to Description of Fusion-fission Dynamics in Super-heavy Element Formation

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A new mechanism of the fusion-fission process for a heavy nuclear system is proposed, which takes place in the (A_1, A_2) space, where A_1 and A_2 are two nuclei, surrounded by a certain number of shared nucleons ΔA. The nuclei A_1 and A_2 gradually lose (or acquire) their individualities with increasing (or decreasing) a number of collectivized nucleons ΔA. The driving potential in the (A_1, A_2) space is derived, which allows the calculation of both the probability of the compound nucleus formation and the mass distribution of fission and quasi-fission fragments in heavy ion fusion reactions. The cross sections of super-heavy element formation in the "hot" and "cold" fusion reactions have been calculated up to $Z_{\rm CN} = 118$.

1. Introduction

The interest in the synthesis of super-heavy nuclei has lately grown due to the new experimental results[1,2] demonstrating a real possibility of producing and investigating the nuclei in the region of the so-called "island of stability". The new reality demands a more substantial theoretical support of these expensive experiments which will allow a more reasonable choice of fusing nuclei and collision energies as well as a better estimation of the cross sections and unambiguous identification of evaporation residues (ER). Unfortunately, at present it is quite difficult (and hardly possible) to make an accurate qualitative analysis of the complex dynamics of the heavy ion fusion reaction leading to the formation in the exit channel of ER of easily fissile super-heavy nucleus.

A whole process of super-heavy nucleus formation is divided usually into three reaction stages even if connected with each other but treated and calculated separately: (1) overcoming the Coulomb barrier and approaching the point of contact $R_{\rm cont} = R_1 + R_2$, (2) formation of the compound mono-nucleus, (3) decay ("cooling") of the compound nucleus. Different theoretical approaches are used for analyzing all the three reaction stages. However, the dynamics of the intermediate stage of the compound nucleus formation is the most vague. It is due to the fact that in the fusion of light and medium nuclei, in which the fissility of the compound nucleus is not very high, the colliding nuclei having overcome the Coulomb barrier form a compound nucleus with a probability $P_{\rm CN} \approx 1$. Thus, this reaction stage does not influence the yield of ER at all. However, in the fusion of heavy nuclei it is the fission channels (regular and quasi-fission) that substantially determine the dynamics of the whole process; the $P_{\rm CN}$ value can be much smaller than unit, while its accurate calculation is very difficult. Moreover, today there are no consensus for the mechanism of the compound nucleus formation itself, and quite different, sometimes opposite in their physics sense, models are used for its description.[3–5]

The production cross section of a cold residual nucleus C, which is the product of neutron evaporation and γ emission from an excited compound nucleus C^*, formed in the fusion process of two heavy nuclei $A_1 + A_2 \to C^* \to C + xn + N\gamma$ at center-of-mass energy close to the Coulomb barrier in the entrance channel, can be decomposed over partial waves and written in the following form

$$\sigma^{xn}_{\rm ER}(E) \approx \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)\cdot\int_0^{\infty} f(B)P^{HW}(B,l,E) \times P_{\rm CN}(B,l,E^*)dB\cdot W_{xn}(l,E^*)\,. \quad (1)$$

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Here $P^{HW} = \left[1+\exp\left(\frac{2\pi}{\hbar\omega(l)}[B+\frac{\hbar^2 l(l+1)}{2\mu R_B^2(l)} - E]\right)\right]^{-1}$ is the penetration probability of the one-dimensional potential barrier given by the usual Hill-Wheeler formula[7] with the barrier height modified to include a centrifugal term. $f(B)$ is the "barrier distribution function",[8] which takes into account the multi-dimensional character of the realistic barrier given by dynamic deformations of nuclear surfaces and/or different orientations of statically deformed colliding nuclei. Integration over effective barrier B means, in fact, integration over such dynamic deforma-

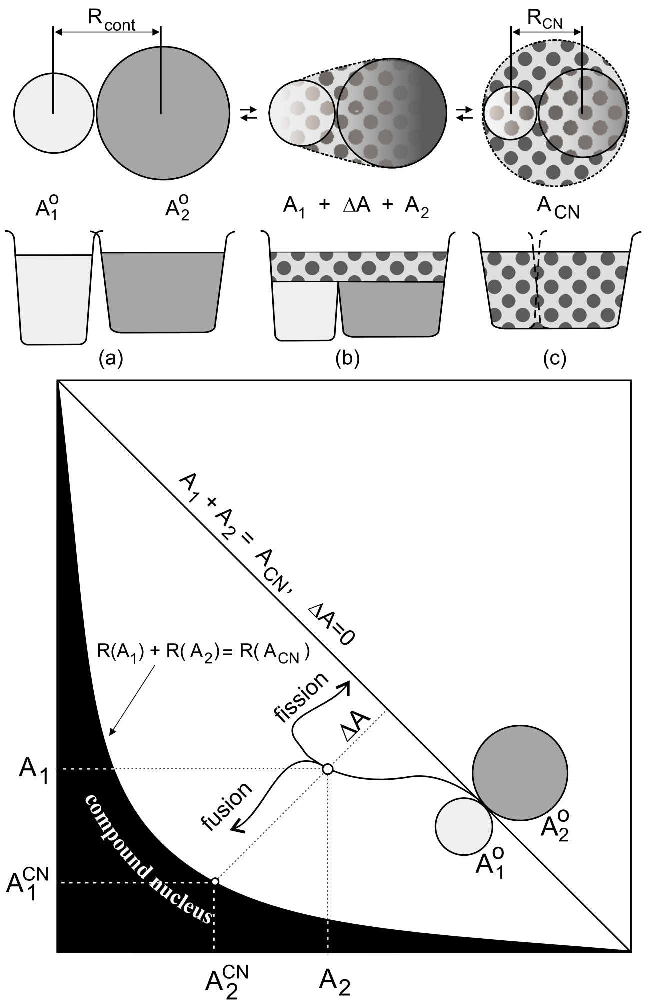

Figure 1. Schematic view of the process of compound nucleus formation, fission and quasi-fission in the space of A_1, A_2, and ΔA, i.e., the number of nucleons in the projectile-like nucleus, target-like nucleus, and shared nucleons, here $A_1 + A_2 + \Delta A = A_{\rm CN}$.

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tions and/or different orientations. A choice of the function $f(B)$, which defines the capture cross section, is discussed in References 5, 6. P_{CN} is the probability that the nuclear system will evolve from a configuration of two touching nuclei into a spherical or nearly spherical form of the compound mono-nucleus. In the course of this evolution the heavy system may, in principle, fall again into two fragments without forming the compound nucleus (quasi-fission) and, thus, $P_{CN} \leq 1$. This probability depends also on initial deformations and orientations of two touching nuclei. The last term in eq 1, W_{xn}, defines the probability of producing the cold evaporation residue C in the process of the compound nucleus decay. It has the initial excitation energy $E^* = E - Q_{gg}^{\text{fus}}$, where E is the beam energy in the center-of-mass system, $Q_{gg}^{\text{fus}} = B(A_1) + B(A_2) - B(C)$, and $B(C)$, $B(A_1)$, $B(A_2)$ are the binding energies of the nuclei. Details of the calculation of survival probability $W_{xn}(l, E^*)$ and discussion of the factors, which bring major uncertainty into estimation of this quantity for super-heavy niclei, can be found in Reference 6. Putting $W_{xn} = 1$ in eq 1 we get the "fusion cross section", putting in addition $P_{CN} = 1$ we get the "capture cross section".

2. Fusion-fission Dynamics of Heavy Nuclei

The following mechanism have been proposed for the compound nucleus formation and quasi-fission process.[5]
(1) Down to the instant of touch the nuclei keep their individualities and the potential energy of their interaction is defined in a usual manner, e.g. by the proximity forces. The distance of contact R_{cont} is smaller by 1–3 fm than the radius of the Coulomb barrier and, thus, the nuclei have to overcome this barrier to reach it.

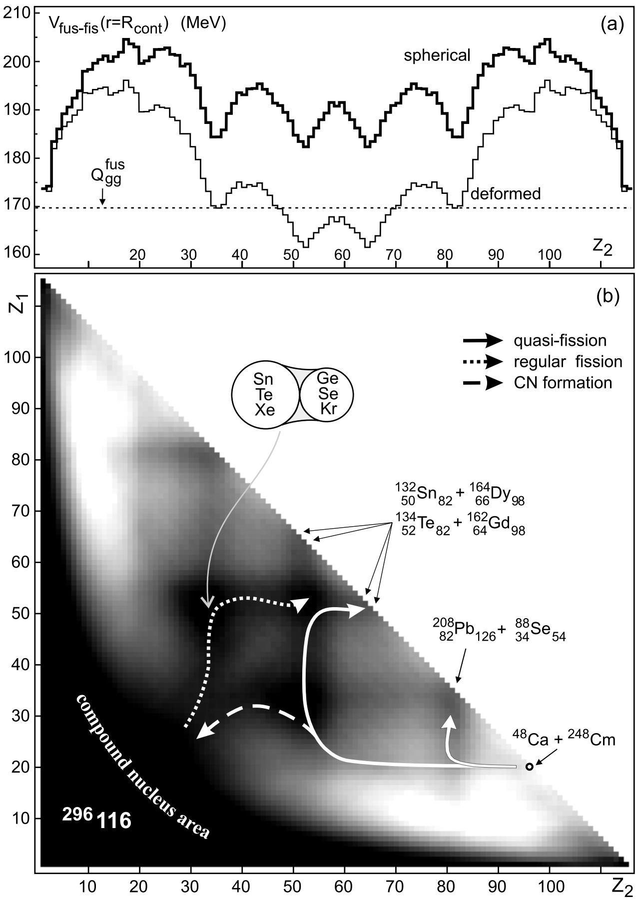

Figure 2. Driving potential $V_{\text{fus-fis}}(Z_1, Z_2)$ of the nuclear system consisting of 116 protons and 180 neutrons. (a) Potential energy of two touching nuclei at $A_1 + A_2 = A_{CN}$, $\Delta A = 0$, i.e., along the diagonal of the lower figure. The thick line corresponds to the case of spherical nuclei, whereas the thin line corresponds to $\delta = \delta_{sd}/2$ (see the text). (b) Topographical landscape of the driving potential on the plane (Z_1, Z_2) (zero deformations). The dark regions correspond to the lower potential energies (more compact configurations).

(2) In the point of contact the nuclei begin to lose their individualities due to an increasing number of shared nucleons ΔA, here $A_1 + A_2 + \Delta A = A_{CN}$ (configuration (b) in Figure 1). Interaction of two touching nuclei A_1 and A_2 weakens with increasing the number of shared nucleons ΔA, and their specific binding energies approach specific binding energy of the compound nucleus. Collectivized nucleons move in the whole volume occupied by the two nuclei and have the average over A_1 and A_2 specific binding energy.
(3) The processes of compound nucleus formation and quasi-fission occur in the space (A_1, A_2), here the compound nucleus is finally formed when two fragments A_1 and A_2 go in its volume, i.e., at $R(A_1) + R(A_2) = R_{CN}$ or at $A_1^{1/3} + A_2^{1/3} = A_{CN}^{1/3}$ (configuration (c) in Figure 1).

The total potential energy of the nuclear system can be defined in the following way

$$\begin{aligned} V_{\text{fus-fis}}(r; Z_1, N_1, Z_2, N_2; \delta_1, \delta_2) \\ = V_{12}(r, \delta_1, \delta_2, \Delta A) - [\tilde{\beta}_1 A_1 + \tilde{\beta}_2 A_2 + \tilde{\beta}\Delta A] \\ + B(A_1^0) + B(A_2^0). \end{aligned} \tag{2}$$

Here $B(A_1^0)$ and $B(A_2^0)$ are the binding energies of the projectile and target; $\tilde{\beta}_1$, $\tilde{\beta}_2$, and $\tilde{\beta} = (\tilde{\beta}_1 + \tilde{\beta}_2)/2$ are the specific binding energies of the nucleons in the fragments A_1, A_2, and that of the shared nucleons ΔA, respectively. These quantities depend on the number of shared nucleons. If we define the measure of the collectivization as $x = \frac{\Delta A}{\Delta A_{CN}}$ (see Figure 1), then $\tilde{\beta}_{1,2}$ can be approximated as $\tilde{\beta}_{1,2} = \beta_{1,2}^{\text{exp}}(1 - x) + \beta_{CN}^{\text{exp}} x$. δ_1 and δ_2 are the dynamic deformations of the fragments. The interaction V_{12} (sum of the Coulomb and nuclear potential) is defined in usual way at $r \geq R_{\text{cont}}$ and gradually goes to zero at $r \geq R_{CN}$ ($x = 1$), see details in Reference 5. Thus, once the compound nucleus has been formed (the dark area in Figure 1), the total energy of the system $V_{\text{fus-fis}} = Q_{gg}^{\text{fus}}$, as it should be if the energy of two resting at infinity initial nuclei is taken as zero. A microscopic calculation within the two-center shell model should be done to find the quantities $\tilde{\beta}_{1,2}$ and V_{12} at $\Delta A \neq 0$. However it is quite difficult to perform such realistic calculations especially for the "contact" configurations with $\Delta A \sim 0$.

The topographical landscape of the driving potential $V(Z_1, Z_2; \delta_{1,2} = 0)$ for the case of formation of the compound nucleus $^{296}116$ is shown in Figure 2. One can see that the shell structure, clearly revealing itself in the contact of two nuclei, i.e.

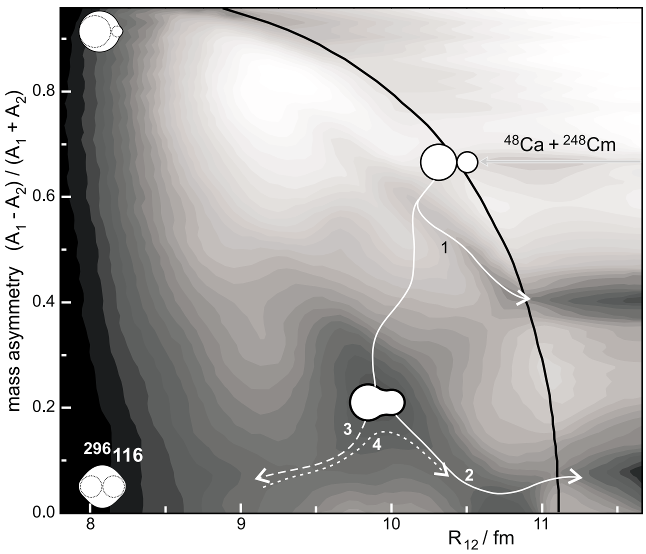

Figure 3. Driving potential $V_{\text{fus-fis}}$ as a function of mass asymmetry and distance between centers of two nuclei with zero deformations. The black solid curve corresponds to the contact configurations. The ways 1 and 2 lead to the asymmetric and near-symmetric quasi-fission channels, whereas 3 and 4 correspond to the compound nucleus formation and its regular fission, respectively. See a conformity with Figure 2.

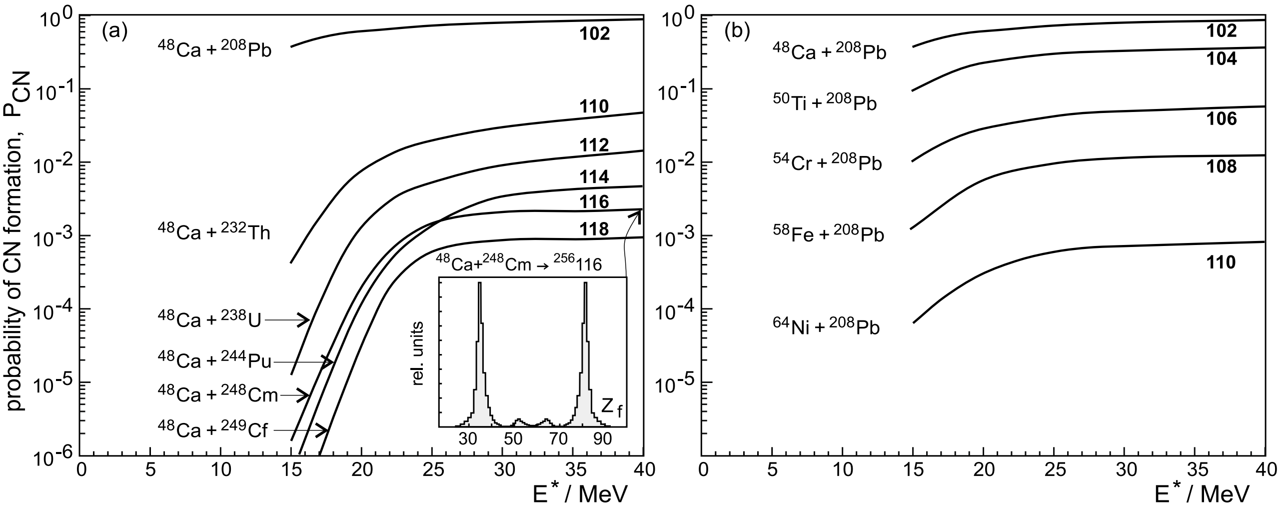

Figure 4. Probability of the compound nucleus formation for the (a) “hot” and (b) “cold” fusion reactions. In the inset the charge distribution of quasi-fission fragments is shown for the $^{48}Ca + ^{248}Cm$ fusion reaction at $E^* = 40$ MeV (linear scale, relative units). The main peaks correspond to the way 1 in Figure 3, whereas the small near-symmetric peaks correspond to the way 2.

at the borderline $A_1 + A_2 = A_{CN}$, (Figure 2(a)) is also retained at $\Delta A \neq 0$ (see, e.g., the deep minima in the regions of $Z_{1,2} \sim 50$ and $Z_{1,2} \sim 82$ in Figure 2(b)). At the synthesis of the nucleus $^{296}116$ in the fusion reaction $^{48}Ca + ^{248}Cm$, after the contact the system decays with a large probability into the quasi-fission channels (mainly asymmetric: Se + Pb, Kr + Hg and also near-symmetric: Sn + Dy, Te + Gd) — solid arrow lines in Figure 2(b). Only a small part of the incoming flux reaches a compound nucleus configuration (dashed arrow line).

In fact, the driving potential (2) is a continuous function at $r = R_{cont}$ and can be used to describe the evolution of the system at all reaction stages. In Figure 3 the driving potential is shown as a function of mass asymmetry and distance between centers of two nuclei. In these variables the driving potential (2) can be easily compared with the potential energy calculated within the two-center shell model. Such comparison have been made in Reference 5 demonstrating a reasonable agreement between the two approaches.

3. Compound Nucleus Formation and Cross Sections of Super-heavy Element Production

Using the driving potential $V_{fus-fis}(Z_1, N_1, Z_2, N_2, \delta_1, \delta_2)$ we can determine the probability of the compound nucleus formation $P_{CN}(A_1^0 + A_2^0 \to C)$, being part of expression (1) for the cross section of the synthesis of super-heavy nuclei. It can be done, for example, by solving the master equation[9] for the distribution function $F(\vec{y} = \{Z_1, N_1, Z_2, N_2, \delta_1, \delta_2\}; t)$. The probability of the compound nucleus formation is determined as an integral of the distribution function over the region $R_1 + R_2 \leq R_{CN}$. Similarly one can define the probabilities of finding the system in different channels of quasi-fission, i.e., the charge and mass distribution of fission fragments measured experimentally. In fact, it is not so easy to perform such realistic calculations due to a large number of the variables.

The master equation approach with restricted number of the variables was used for a rough estimation of P_{CN} and evolution of the nuclear system in the (Z_1, Z_2)-space. First, the deformations of the fragments were fixed at $\delta = \delta_{sd}/2$, where δ_{sd} is the deformation corresponding to the saddle of the interaction potential V_{12} in the (r, δ)-space, see Reference 5. Then the potential energy was minimized over N_1 and N_2 and a two-dimensional driving potential $V_{fus-fis}(Z_1, Z_2)$ was calculated. Finally we solved the master equation for the distribution function $F(y = \{Z_1, Z_2\}, t)$

$$\frac{\partial F}{\partial t} = \sum_{y'} \lambda(y, y') F(y', t) - \lambda(y', y) F(y, t) . \quad (3)$$

We used the same macroscopic transition probabilities as in Reference 9, i.e., $\lambda(y, y') \sim \exp\{[V_{fus-fis}(y') - V_{fus-fis}(y)]/2T(y)\}$, where $T = \sqrt{[E_{cm} - V_{fus-fis}(y)]/a}$ is the local temperature and a is the level density parameter. Equation 3 describes an overdamped evolution, when a potential energy of the nuclear system plays a major role. The sum over y' in eq 3 is extended only to nearest configurations $Z_{1,2} \pm 1$ (no fragment transfer). Equation 3 was solved up to the moment when the total flux comes to the compound nucleus configurations (dark area in Figure 2(b)) and/or escapes into the fission channels, giving us the probability of the compound nucleus formation and the charge distribution of quasi-fission fragments.

Results of such calculations are shown in Figure 4. For the “hot” fusion reactions, based on using ^{48}Ca as a projectile, the probability of the compound nucleus formation falls down very sharply at first with increasing Z_{CN}, but then it remains at the level of 10^{-3} for $Z_{CN} = 114$–118 at excitation energies $E^* > 30$ MeV. Such behaviour of P_{CN} reflects the fact of insignificant changes of $V_{fus-fis}(A_1, A_2)$ for all these reactions. In contrast with that, for the “cold” fusion reactions, based on using ^{208}Pb as a target, the probability of the compound nucleus formation decreases very fast with increasing Z_{CN}.

Calculating the capture cross sections and the survival probability W_{xn} within the approach proposed in References 5, 6 and

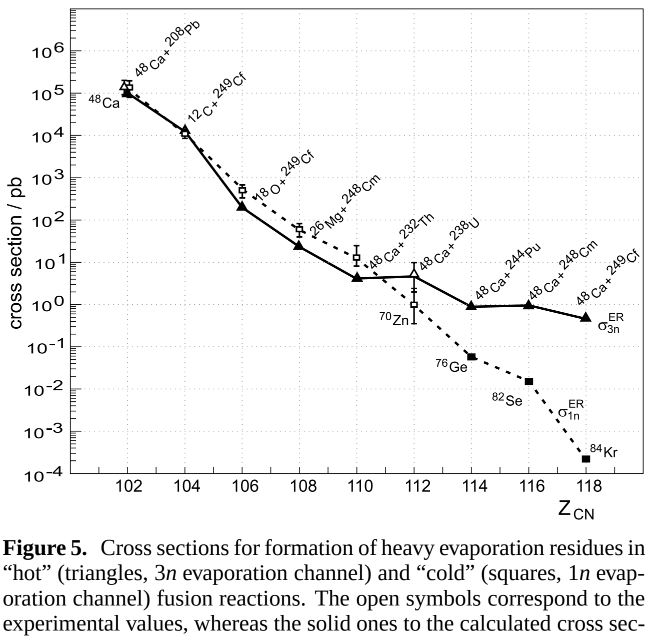

Figure 5. Cross sections for formation of heavy evaporation residues in “hot” (triangles, 3n evaporation channel) and “cold” (squares, 1n evaporation channel) fusion reactions. The open symbols correspond to the experimental values, whereas the solid ones to the calculated cross sections. For the “hot” fusion reactions the corresponding projectile-target combinations are shown. For the “cold” fusion reactions ^{208}Pb is used as a target and only the projectiles are shown.

using the fission barriers based on the ground state shell corrections of Möller et al.,[10] we estimated the cross sections of super-heavy element formation in the "hot" fusion reactions leading to heavy nuclei with $Z_{CN} \geq 102$ — Figure 5. The cross sections for formation of super-heavy nuclei with $Z = 114$–118 in the 3n and 4n evaporation channels of the "hot" fusion reactions were found to be at the level of 0.1–1 pb.

For the available experimentally "cold" fusion reactions the cross sections for formation of the same elements in the 1n evaporation channel are much lower, see Figure 5. A gain of about three orders of magnitude in the survival probability — $W_{1n}(E^* \approx 15 \text{ MeV})/W_{3n}(E^* \approx 35 \text{ MeV}) \sim 10^3$ — is compensated here by a loss of 2 orders in the capture cross sections and more than 2 orders of magnitude in the probability of the compound nucleus formation.

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