A Dynamical Approach to Heavy-ion Fusion: ^{48}Ca + ^{244}Pu

G. I. Kosenko,[*,a,b] C. Shen,[b,c] and Y. Abe[d]

[a]*Department of Theoretical Physics, Omsk State University, Omsk 644077, Russia*
[b]*RIKEN, Wako-shi, 351-0198, Japan*
[c]*Department of Nuclear Physics, China Institute of Atomic Energy, P.O.Box 275(18), Beijing 102413, P.R. China*
[d]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

Received: February 18, 2002; In Final Form: April 30, 2002

We developed a combined approach for the calculation of the fusion cross section and the evaporation residue cross sections in reactions with heavy ions. The method consists of two parts. The first part is a Langevin dynamics with which we can calculate the touch probability of the incident ions $P_{\rm touch}$ and the formation probability of the compound nucleus $P_{\rm form}$. Their product gives the fusion probability $P_{\rm fus}$. The second part is a statistical calculation. We employ the code HIVAP for calculations of the survival probability $P_{\rm surv}$ of the compound system. The approach is tested on the reaction ^{48}Ca + ^{238}U and is applied to ^{48}Ca + ^{244}Pu system. The preliminary results are presented.

1. Introduction

During last decade a big progress has been achieved in synthesis of the super heavy elements (SHE). Events which indicate syntheses of the elements 114 and 116 have been observed.[1] But the experiments have been made more or less in empirical ways. Thus, theoretical efforts are being waited for to provide suggestions effective for planning of experiments. For that purpose most urgent to understand is mechanisms of fusion in massive systems where so-called fusion hindrance is experimentally known to exist.[2] One of ways to this direction is a development of the two-step approach for fusion,[3] which is expected to enable us to calculate fusion probabilities for superheavy compound nuclei, properly taking into account the possible mechanisms for the hindrance. In the present paper, we explain the theoretical framework and present preliminary results on ^{48}Ca + ^{244}Pu system.

According to the theory of the compound nucleus, residue cross sections are given by a product of fusion probability $P_{\rm fus}$ and survival probability $P_{\rm surv}$, which are independent with each other except conserved quantities such as energy, total angular momentum, etc.;

$$\sigma_{\rm res} = \pi\lambda^{-2}\sum_J (2J+1)\cdot P^J_{\rm fus}(E_{\rm c.m.})\cdot P^J_{\rm surv}(E_{\rm ex}), \qquad (1)$$

where the excitation energy of the compound nucleus $E_{\rm ex}$ is equal to the sum of the *c.m.* incident energy $E_{\rm c.m.}$ and the Q value of the fusion reaction. The probability of compound nucleus formation is given by the product of two probabilities which correspond to two successive processes, respectively;

$$P_{\rm fus} = P_{\rm touch}\cdot P_{\rm form} \qquad (2)$$

The first factor $P_{\rm touch}$ is the probability for the nuclear matters of the incident ions to touch each other, and the second $P_{\rm form}$ is the probability of compound nucleus formation, starting from the pear-shaped composite system made of the incident ions. The both processes are described by Langevin equations, though the first one also describes a heating-up process of the system.[4,5] Since the two processes are successive, it should be noticed here that the results obtained in calculations of the first process are used as input data of initial conditions for calculations of the second process. Combined with the survival probability $P_{\rm surv}$ calculated with the statistical decay part of HIVAP,[6] we obtain final residue cross sections for SHE according to eq 1.

*Corresponding author. E-mail: kosenko@nrsun.jinr.ru.

2. Entrance Channel

Dynamical calculations of collision processes in the entrance channel provide us with $P_{\rm touch}$. We employ the classical trajectory model to describe collision dynamics of the entrance channel. The spherical shapes of the ions are assumed not to change for simplicity. We introduce variables with the intrinsic spins L_1 and L_2 of the incident ions, respectively,

$$L^+ = L_1 + L_2 = L_{in} - L(t)$$

and

$$L^- = \frac{C_1L_2(t) - C_2L_1(t)}{C_1 + C_2}$$

as in Reference 7. Here C_1 and C_2 are approximately equal to ions radii R_i ($R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$ fm) and are given as $C_i = R_i(1-(b/R_i)^2)$, $i = 1,2$, $b = 1$ fm. And we have $dL^+/dt = -dL(t)/dt$. Therefore Langevin equations in the present model are given:

$$\frac{dr}{dt} = \frac{1}{\mu}p\,, \qquad (3)$$

$$\frac{dp}{dt} = -\frac{dV}{dr} - \gamma_r\frac{p}{\mu} + \omega_r(t)\,, \qquad (4)$$

$$\frac{d}{dt}\begin{pmatrix} L \\ L^- \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}\begin{pmatrix} L \\ L^- \end{pmatrix} + \begin{pmatrix} \tilde{\gamma_{11}} \\ \gamma_{21} \end{pmatrix} L_{in} + \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix}\begin{pmatrix} \omega \\ \omega^- \end{pmatrix}, \qquad (5)$$

where V is a sum of the Coulomb energy, of the rotational energy, and of the nuclear energy — V_N. And

$$\begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} = \begin{pmatrix} -\Psi(r)c_T\left[\frac{1}{m} + \frac{r^2}{(C_1+C_2)^2}\frac{C_1^2J_2+C_2^2J_1}{J_1J_2}\right] & -\Psi(r)c_T\left[\frac{r^2}{(C_1+C_2)}\frac{C_1J_2-C_2J_1}{J_1J_2}\right] \\ -\Psi(r)c_{\rm roll}g^2\left[\frac{1}{(C_1+C_2)}\frac{C_1J_2-C_2J_1}{J_1J_2}\right] & -\Psi(r)c_{\rm roll}g^2\left[\frac{J_1+J_2}{J_1J_2}\right] \end{pmatrix}$$

with γ_{ij} being the friction tensor of the system. $\tilde{\gamma_{11}} = \gamma_{11} - \Psi(r)c_T/m$, and $\theta_{ij} = \theta_{ij}(\gamma_{ij})$ define magnitudes of random force. $\omega_i(\omega, \omega^-, \omega_r)$ are random numbers with properties

$$\langle\omega_i\rangle = 0, \quad \langle\omega_i(t)\omega_j(t')\rangle = 2\delta_{ij}\,\delta(t-t'). \qquad (6)$$

The function $\Psi(r)$ defines a radial form factor of the friction of the system; $\Psi(r)\,c_T$ — the tangential friction and $\Psi(r)\,c_{\rm roll}\,g^2$ ($c_{\rm roll} = const\cdot c_T$) — the rolling friction, and $\gamma_r = 2c_T$ — the radial friction.

Published on Web 6/30/2002

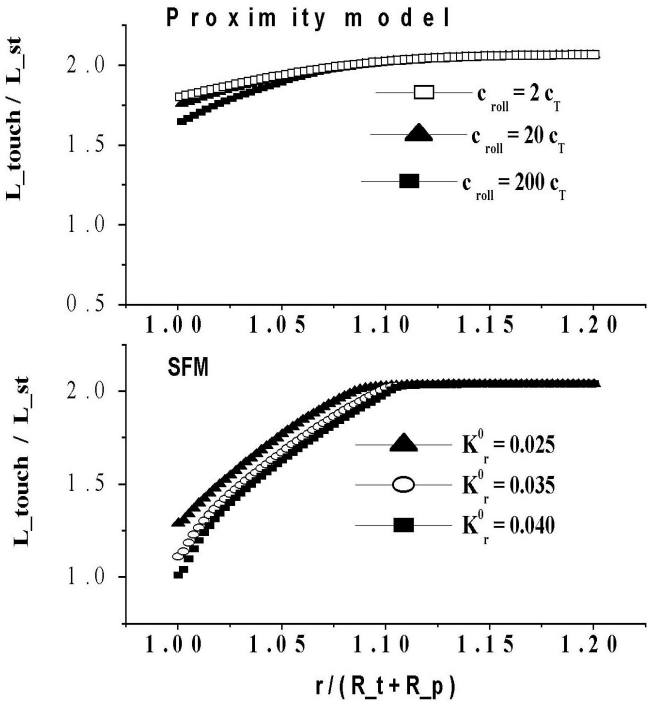

Figure 1. The dependence of reaching the sticking limit on friction forces The relation the average angular momentum in touching point "L_touch" to the sticking limit angular momentum "L_st" versus relative distance (R_t and R_p are radii of the target and the projectile, respectively) in the dependence of the friction forces: top — proximity model, bottom — SFM.

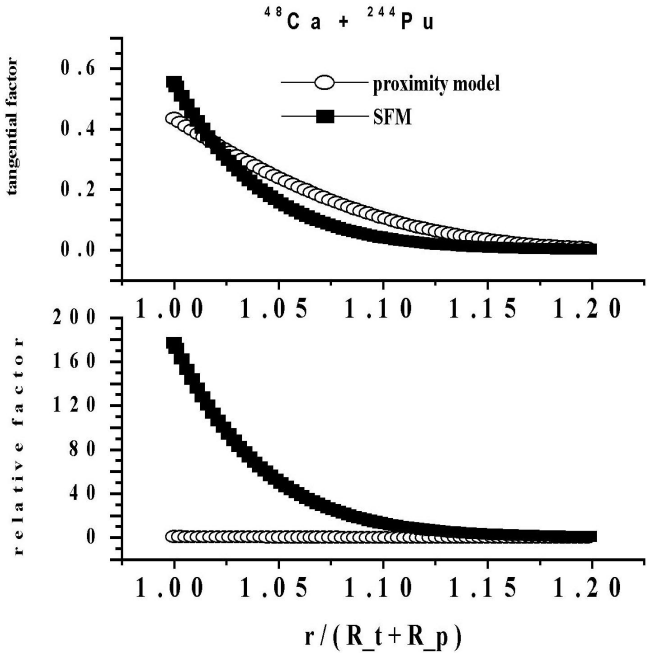

Figure 2. The comparison of the tangential (top) and the relative (bottom) friction form factors between SFM and proximity model versus relative distance, the same as Figure 1.

We investigate two types of friction in the entrance channel. The above forms of γ_{ij} connect with the proximity friction and the proximity potential energy V_N (Ref. 8) for colliding ions. In the proximity friction model the friction is just stemming from exchanges of nucleons between slowly moving ions, which is the window formula part of so-called one-body wall-and-window formula used for analyses of fission. It is not so strong as the surface friction model (SFM), which is introduced by Gross and Kalinowski.[9] The rolling friction is not included in SFM. In the case only with the radial and the tangential frictions, we have the relation $\Psi(r)\, c_T = K_\phi^0 \left(\frac{dV_N}{dr}\right)^2$ for the tangential friction and $\gamma_r = K_r^0 \left(\frac{dV_N}{dr}\right)^2$ for the radial friction, where V_N is the

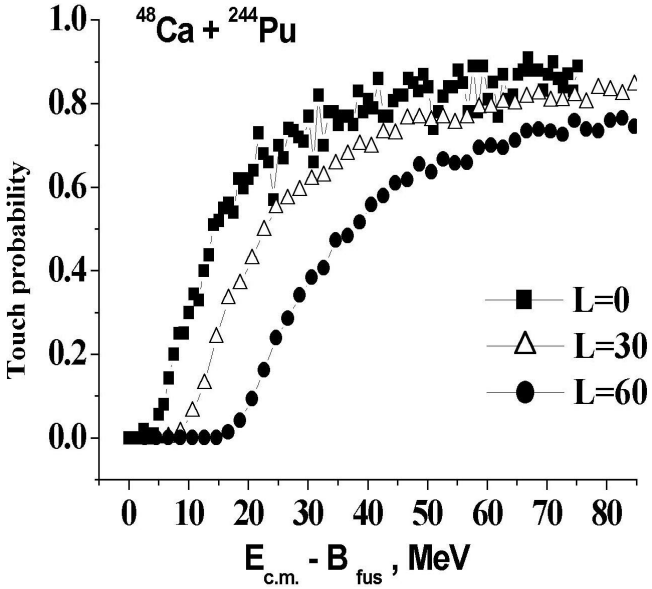

Figure 3. The touch probability for different initial angular momenta versus $E_{\text{c.m.}} - B_{\text{fus}}$.

nuclear potential energy[9] between the ions. As will be seen below, SFM is of very strong frictional forces. Therefore, we have two extreme types of friction: strong (SFM) and weak (proximity). Which one is preferable? Or more realistic? In order to answer this question, we need to make a systematic analysis.

We expect the incident ions to be united when the ions touch. Then, the system moves (rotate) as a whole. The target and the projectile have the same angular velocity as that of the whole system. This situation is called the sticking limit.[7] Therefore, we investigate the both friction mechanisms in reference to the sticking limit. Figure 1 shows our results. We calculate the quantity $L_{\text{touch}}/L_{\text{st}}$, which is equal to 1 if the system reaches the sticking limit. First, we investigate both model with their standard parameters. We used $c_{\text{roll}} = 2c_T$ for the proximity friction, and $K_\phi^0 = 0.0001$, $K_r^0 = 0.035$ in unit of 10^{-21} s/MeV for SFM. The proximity friction can not give rise to the sticking limit. Actually, the system is far from the sticking limit in the touching point. A possible way to make the proximity friction stronger is to increase *const* in the rolling friction (see page 19), while the other parts are calculated according to the prescription. We increase $const \cdot c_{\text{roll}}$ two orders of magnitude, but still the sticking limit is not realised.

The opposite situation is obtained with SFM. The system is very near to the sticking limit with the standard parameters. (By the way, the authors[9] gave two values for the parameter K_r^0. They are 0.035 and 0.040. The second one gives the quantity $L_{\text{touch}}/L_{\text{st}}$ is almost exactly equal to 1.) The main difference between two models is in the radial friction as shown in Figure 2. The tangential friction of SFM is stronger than that of the proximity by 30%. The radial friction in SFM is stronger than proximity one by two orders of magnitude. It means that the sticking limit depends first of all on the radial friction of the system. And the second conclusion from this figure would be that the friction between two ions is due to more complicated mechanisms than nucleon exchanges between colliding ions alone. We, thus, employ SFM for the entrance channel below. The touch probability P_{touch}, of course, depends on the angular momentum. We calculate P_{touch} for a few initial angular momenta. The results are shown in Figure 3. We can see that the touch probability slightly decreases, as the angular momentum increases. The value $P_{\text{touch}} = 1/2$ is reached when the *c.m.* energies become higher than the fusion barrier by 15, 25, 35 MeV for $L = 0$, 30, $60\hbar$, respectively. It means that the extra push energy increases with the angular momentum. The grazing angular momentum which is the maximum angular momentum reaching the barrier in case of no friction for a given $E_{\text{c.m.}}$ ($V(L_{\text{gr}}) = E_{\text{c.m.}}$) dissipates

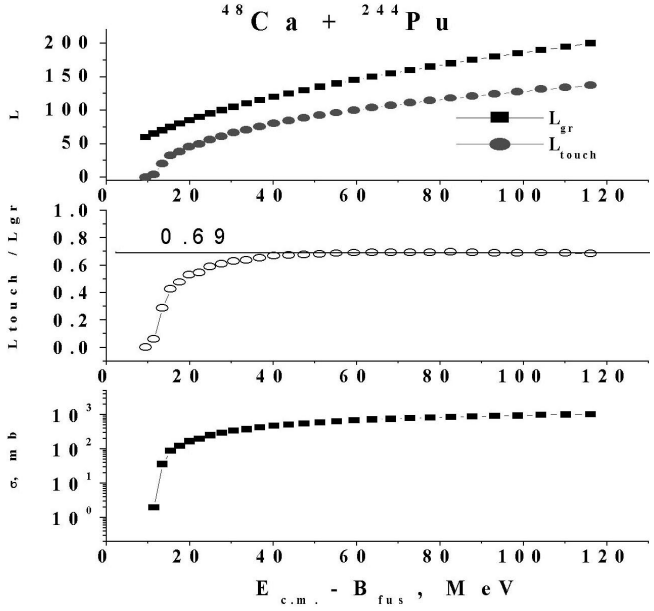

Figure 4. The L_{gr} and L_{touch} — top, the quantity L_{touch}/L_{gr} — middle, and the touch cross section — bottom versus $E_{c.m.} - B_{fus}$ for the reaction $^{48}Ca + ^{244}Pu$.

as the ions approach each other. Figure 4 shows decreases of the orbital angular momentum, that is, from L_{gr} to L_{touch}. The quantity L_{touch}/L_{gr} increases from zero up to constant value $\sim$0.7, as $E_{c.m.}$ increases. The limiting value does not depend on ion combinations.

3. Evolution of Shape and Survival Probability

The dynamical collision processes analysed in the previous section give us the united system consisting of two touching ions. Then, the next step is to obtain the probability P_{form} for the system to reach the spherical shape. In order to describe the evolution of this system to the spherical shape, we employ a Langevin equation for the variables describing collective motions for which we use the parameters of the two-center model. The equation is slightly different from one used in previous section,

$$\begin{aligned} \frac{dq_i}{dt} &= \left(m^{-1}\right)_{ij} \cdot p_j, \\ \frac{dp_i}{dt} &= -\frac{\partial U}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} \cdot p_j p_k \\ &\quad -\gamma_{ij}\left(m^{-1}\right)_{jk} \cdot p_k + g_{ij}\,\omega_j(t), \end{aligned} \tag{7}$$

where U is the liquid drop model (LDM) potential energy, m_{ij} and γ_{ij} are the shape-dependent collective inertia mass and friction tensors, respectively. $\omega_i(t)$ is normalized random force with properties as defined in eq 6. The strength of the random force g_{ij} is related to the friction; $\gamma_{ij}T = g_{ik}g_{jk}$. The Werner-Wheeler approximation[10] is used for the inertia tensors. The friction tensor is assumed to be that calculated with the wall-and-window one-body model.[11] Many trajectories are taken into account in order to obtain the formation probability. All trajectories start from the touch point for various angular momenta and relative momenta. Distributions of the relative momentum and the angular momentum have been obtained in the entrance channel calculation. Most of trajectories go back to reseparation, as shown with the thick line in Figure 5. A fraction of them reaches the spherical shape as shown in Figure 5 with thin lines. The latters give distributions of the excitation energy and the angular momentum of the compound nucleus formed. The final stage of the calculation is made by using the statistical code HIVAP in order to obtain the survival probability P_{surv}. As for the parameters in HIVAP, we adopt the standard values recommended by Reisdorf. We show a map of potential energy of the $^{48}Ca + ^{238}U$

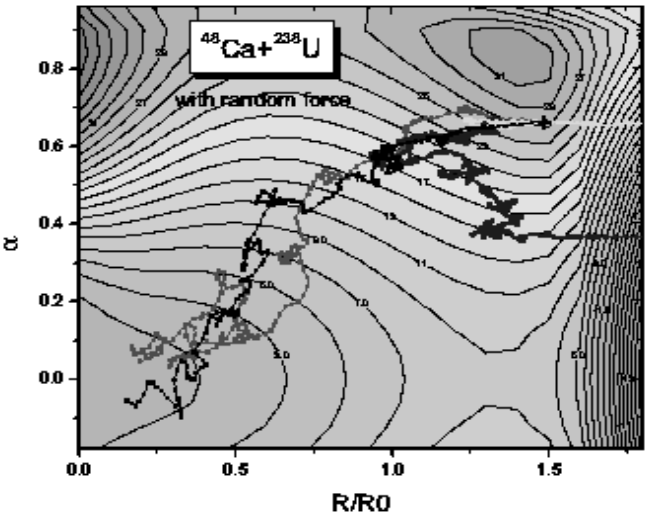

Figure 5. The map LDM of the potential energy. The axes are α — the asymmetry parameter, R — elongation parameter in unit R_0, the radius of the sphere with the same volume. The different lines show examples of the trajectories of shape evolution. The thick lines show the trajectories going back to reseparation and the thin lines show the trajectories which go to the compound nuclei.

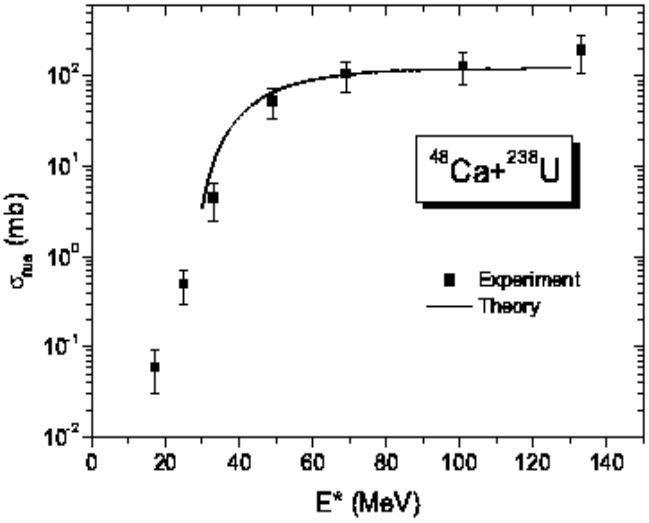

Figure 6. Calculated fusion cross section and corresponding experimental data[12] for $^{48}Ca + ^{238}U$ system.

reaction because this reaction was used as a test reaction. The fusion cross sections were measured for this reaction in Reference 12. This gives us a possibility of testing the present model calculations and of adjusting the parameters. Our results are compared with the experimental data in Figure 6, where we assumed the remaining radial kinetic energy to be about 25% of the maximum available energy at the contact point. With zero remaining energy, the calculated values are smaller than the experiments by factor $1/$several. This means that the friction of SFM might be too strong. Of course, for a reproduction of the fusion data, there is another possibility of reducing the wall-and-window friction[13,14] in the last part of our dynamical calculations. But below, we follow the former way of adjusting friction strengths.

4. Results and Conclusion

We applied the above described formalism to calculations of the fusion cross section and of the evaporation residue cross sections for the reaction $^{48}Ca + ^{244}Pu$. The results are shown in Figure 7. Masses of the heaviest elements are not known experimentally, therefore we employ the table of the mass by Möller et al.[15] Then, the calculations have given unrealistically large evaporation cross sections. In order to obtain a realistic result, we have to decrease the absolute value of the shell correction

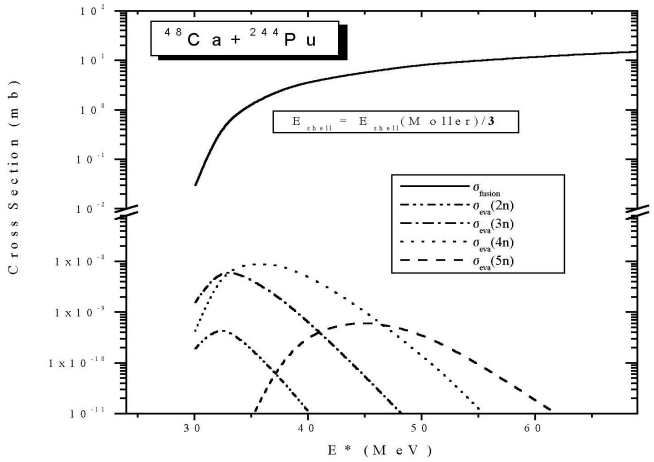

Figure 7. Fusion cross section and cross section of (*xn*)-reaction. The curves are of theoretical calculations. The experimental residue cross section is ∼1 pb and excitation energy of the system is 35 MeV.[1]

energy of Reference 15 with factor $1/3$, which would not be so ridiculous, considering that the available mass (shell correction energy) predictions are different with each other (see for comparison[16]). An assumption about a closed shell is one of the reasons of this discrepancy. A value of the shell correction energy changes if we assume for the closed shell $Z = 114$ or $Z = 126$. The shell correction energy is much smaller in the second case. But, unfortunately, nobody knows the real situation. The change of the mass by 1 MeV gives the change of the cross section by one order, therefore improvements of our knowledge about masses of heaviest elements provide us with reliable calculations of survival probability of the super heavy elements.

We have developed an approach which gives us the possibility to estimate the formation probability of superheavy elements. Already preliminary results obtained with it appear to be reasonable. We have understood that friction in the entrance channel is strong and that the proximity type of the friction is not strong enough for explanation of di-nucleus formation of colliding ions. Our calculations suggest that the shell correction energies for the heavy elements might not be so large as proposed. Of course, there are a few other possibilities which give rise to the similar effects to the reduction of the shell correction energy, for example, the collective enhancement factor for the fission width, so-called a_f/a_n being larger than 1, etc. We hope that this approach provides us with a physical understanding of the mechanism of synthesis of the superheavy elements.

The author (K.G.I.) thanks the Organize Committee for the invitation to give a talk and financial supports. This work was supported partially by RIKEN. The authors thank S. Yamaji and T. Otsuka for warm hospitality in RIKEN. The work is partially supported by the Grant-in-Aid of JSPS (no. 13640278).

References

(1) Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, I. V. Shirokovsky, Yu. S. Tsyganov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, S. Iliev, V. G. Subbotin, A. M. Sukhov, G. V. Buklanov, K. Subotic, M. G. Itkis, K. J. Moody, J. F. Wild, N. J. Stoyer, M. A. Stoyer, and R. W. Lougheed, Phys. Rev. Lett. **83**, 3154 (1999).
(2) S. Bjornholm and W. J. Swiatecki, Nucl. Phys. A **347**, 471 (1982).
(3) Y. Abe, Eur. Phys. J. A **13**, 143 (2002).
(4) Y. Abe, C. Gregoire, and H. Delagrange, J. Phys. (Paris) **47**, c329 (1986).
(5) P. Fröbrich and S. Y. Xu, Nucl. Phys. A **477**, 143 (1988).
(6) W. Reisdorf, *The Fusion Evaporation Code HIVAP*.
(7) C. F. Tsang, Physica Scripta **10A**, 90 (1974); R. Bass, *Nuclear Reaction with Heavy Ions* (Springer-Verlag, Berlin, 1980).
(8) J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. **108**, 427 (1977); J. Randrup, Ann. Phys. **112**, 356 (1978); D. Bangert and H. Freiesleben, Nucl. Phys. A **340**, 205 (1980).
(9) D. H. E. Gross and H. Kalinowski, Phys. Rep. **45**, 175 (1978).
(10) K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C **13**, 2385 (1976).
(11) J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki, Ann. Phys. **113**, 330 (1978).
(12) W. Q. Shen, J. Albinski, A. Gobbi, S. Gralla, K. D. Hildenbrand, N. Herrmann, J. Kuzminski, W. F. J. Müller, H. Stelzer, J. Tōke, B. B. Back, S. Bjørnholm, and S. P. Sørensen, Phys. Rev. C **36**, 115 (1987).
(13) J. R. Nix and A. J. Sierk, Preprint LA-UR-87-133 (Los-Alamos National Laboratory, 1987).
(14) A. V. Karpov, P. N. Nadtochy, D. V. Vanin, and G. D. Adeev, Phys. Rev. C **63**, 054610 (2001).
(15) P. Möller, J. R. Nix, W. D. Myers, and J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
(16) H. Koura, M. Uno, T. Tachibana, and M. Yamada, Preprint RIKEN-AF-NP-304 (2001).