# Alpha-decay Half-lives and Fission Barriers for Superheavy Nuclei Predicted by a Nuclear Mass Formula

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We estimate α-decay half-lives from $Q_\alpha$ values with phenomenological formulas. The parameter values of the formulas are adjusted by using experimental half-lives and $Q$ values of the Evaluated Nuclear Structure Data File (ENSDF) for a wide nuclidic region. The half-lives $T_\alpha$ for unknown nuclei are estimated by this phenomenological formula with the use of $Q_\alpha$ values taken from KUTY mass formula. In addition to the half-lives, we estimate spontaneous fission barriers calculated by the method to obtain the shell energies of KUTY formula. In the superheavy region, the barrier heights of the nuclei near the nucleus $^{304}122$ are about 8 MeV and their widths are fairly wide. Therefore these spontaneous-fission half-lives are expected to be very long. On the other hand, there are some neutron-deficient nuclei whose spontaneous-fission half-lives expected to be rather short because their fission barrier heights are small and the widths of them are narrow.

## 1. Introduction

The productions of superheavy nuclei are recently well performed.[1] In this region, α decay and spontaneous fission are the main decay modes.

Our group has recently constructed a nuclear mass formula,[2,3] which we refer to as KUTY formula, composed of a gross part and a shell part. For the shell part, we first calculate proton and neutron spherical shell energies by using modified Woods-Saxon type potentials, which we have newly constructed.[4] A notable feature of our mass formula is a new method of obtaining shell energies of deformed nuclei. The shell energy of a deformed nucleus is expressed as an appropriate mixture of spherical shell energies added to an average deformation energy. This mass formula gives ground-state masses and shapes for the nuclei ranging from $^4$He to the superheavy nuclei. The standard deviation of the calculated masses from the experimental masses of the 1995 Mass Evaluation[5] is about 680 keV. Using this formula, we estimate α-decay $Q$ values and spontaneous fission barriers.[6] As for the estimation of the α-decay half-lives, we take some phenomenological formulas with some adjustable parameters. As for the spontaneous fission barriers, we calculate the potential energy surface by the method to obtain the shell energies of KUTY formula and then obtain the fission-barrier height.

In sect. 2 we estimate the α-decay half-lives, and in sect. 3 we show the spontaneous fission barrier heights.

## 2. Alpha-decay Half-life

**2.1. Phenomenological Formulas of α-decay Half-lives.** We first estimate the α-decay half-lives $T_\alpha$ (s) from experimental $Q_\alpha$ values (MeV) with a phenomenological relation in a wide nuclidic region. The α-decay half-life is written as

$$T_\alpha = \log_e 2/(N_{\rm coll}\times P)\ , \qquad (1)$$

where $N_{\rm coll}$ is collision frequency of an α particle to a potential wall and $P$ is penetration probability. In the WKB approximation, the probability $P$ for a spherical nucleus is approximately written as

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \{2\frac{m_\alpha m_f}{m_\alpha+m_f}(V(r)-Q_\alpha)\}^{1/2}dr\right], \qquad (2)$$

where $b$ and $R$ are outer and inner radii of α-particle potential $V(r)$ penetrated by α particle with $Q_\alpha$, and $m_\alpha$ and $m_f$ are masses of an α particle and a daughter nucleus, respectively.

Here we consider two phenomenological formulas. One is the Viola-Seaborg formula[7] with an even-odd hindrance term $h$ as

**Formula (A)**

$$\log_{10} T_\alpha = (aZ+b)/\sqrt{Q_\alpha} + (cZ+d) + h\ , \qquad (3)$$

where $a$, $b$, $c$, $d$ are parameters.

Another formula is deduced from the penetration probability neglecting higher order terms as

**Formula (B)**

$$\begin{aligned}\log_{10} T_\alpha \;=\; & 1.7195\sqrt{\frac{A-4}{A}}Z_{\rm D}/\sqrt{Q_\alpha} \\ & -1.2901\sqrt{\frac{A-4}{A}}\sqrt{RZ_{\rm D}} \\ & +0.07466\sqrt{\frac{A-4}{A}}R^{3/2}/Z_{\rm D}^{1/2}Q_\alpha \\ & -\log_{10}N_{\rm coll} - 1.59175 + h\ , \qquad (4)\end{aligned}$$

where

$$R = r_0 A_{\rm D}^{1/3} + d_0\ , \quad r_0 = 1.08 \text{ fm}\ , \qquad (5)$$

where the decimals in eq 4 are obtained when we take a spherical Coulomb potential, the subscript D indicates a daughter nucleus, and $N_{\rm coll}$ and $d_0$ are adjustable parameters.

These parameter values are adjusted with the use of the experimental $T_\alpha$ and $Q_\alpha$ of the Evaluated Nuclear Structure Data File (ENSDF), 2000 August version.[8] We remove the data estimated by systematics or calculations, the data having only upper or lower limits, and the data of $^8$Be from input data. The $Q_\alpha$ of $^{180}$Pb is evaluated as 5.851 MeV in the ENSDF. However, since the original experimental α-particle energy $E_\alpha$ is 7.23 MeV,[9] we take 7.39 MeV $[=E_\alpha\times A/(A-4)]$ as $Q_\alpha$ for $^{180}$Pb. We first adjust the parameters for even-even nuclei. As for odd-$A$ and odd-odd nuclei, we use the same parameters for even-even nuclei and then we determine $h$ so as to reproduce the reasonable half-lives for odd-$A$ and odd-odd nuclei. As a result, we choose a simple expression of $h$ as

$$h = h_0\delta_{\rm eo}\ , \qquad (6)$$

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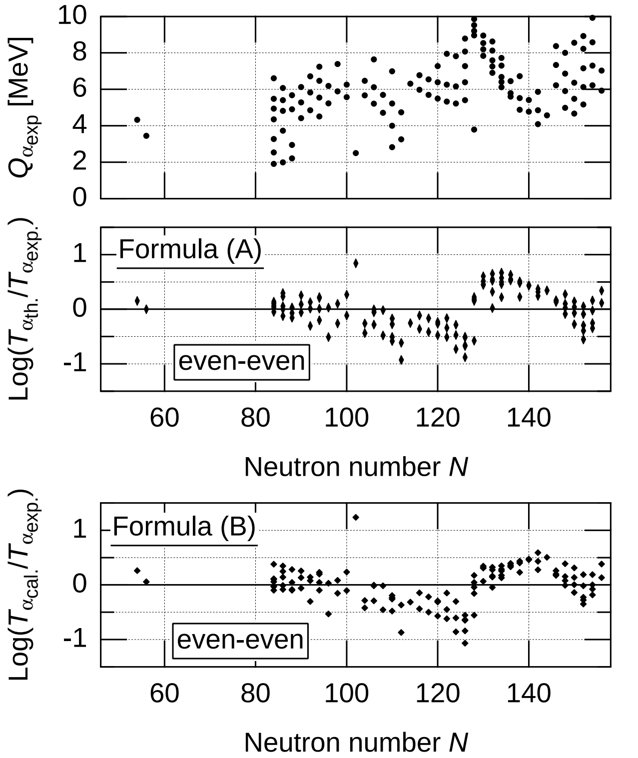


**Figure 1.** Experimental $Q_\alpha$ (upper). Differences between experimental and estimated $\log T_\alpha$ of Formula (A) (middle) and Formula (B) (lower). All data are for even-even nuclei.

with

$$\delta_{eo} = \begin{cases} 0 & \text{for even-even} \\ 1 & \text{for odd-}A \\ 2 & \text{for odd-odd.} \end{cases} \quad (7)$$

Here, $h_0$ is taken from the average of differences of experimental half-lives from estimated ones (as $h = 0$) for odd-$A$ nuclei.

The results for two formulas are in the following.

**Formula (A)**

The values of parameters are $a = 1.55261$, $b = 0.73247$, $c = -0.21669$, $d = -31.9949$, and $h_0 = 0.56718$ for $Q_\alpha$ and $T_\alpha$ in MeV and second, respectively. The root-mean square (RMS) deviation of $\log T_\alpha$ from experimental ones of 120 even-even nuclei is 0.3625. The RMS deviation is 0.7708 for 151 odd-$A$ nuclei and is 0.9845 for 63 odd-odd nuclei. Although $10^{-d}$ roughly corresponds to the collision frequency $N_{coll}$ of the α particle which should be about $10^{20-22}$, the above absolute value of $d$ seems to be too large. In Figure 1 (the upper and middle parts), we show the experimental $Q_\alpha$ and the differences between the experimental and estimated $T_\alpha$ with the use of Formula (A).

**Formula (B)**

The values of fitted parameters are $N_{coll} = 10^{20.05}$, $d_0 = 2.0$ fm, and $h_0 = 0.61410$ for $Q_\alpha$ and $T_\alpha$ in MeV and second, respectively. The values of $N_{coll}$ and $d_0$ are within reasonable values. The RMS deviation of $\log T_\alpha$ for 120 even-even nuclei is 0.3512. In Figure 1 (the lower part), we show the differences between the experimental and estimated $T_\alpha$ with the use of Formula (B). In the region $126 \le N \le 142$, the discrepancy of Formula (B) is reduced in comparison with one of Formula (A). Both of the middle and lower figures show distinct discontinuities at $N = 126$ because of the magicity. At $N = 102$ ($^{174}Hf_{102}$), large discrepancies are also seen. This nucleus is located on the vicinity of β-stability line and is isolated from the other even-even nuclei on the $N$-$Z$ plane and have relatively larger deformation than the others. We show the differences between the experimental and estimated $T_\alpha$ for odd-$A$ and odd-odd nuclei in Figure 2. The RMS deviation is 0.7500 for 151 odd-$A$ nuclei, and is 0.9802 for 63 odd-odd nuclei.

**2.2. Estimation in the Superheavy Region.** In order to compare the above two formulas, we show the experimental and

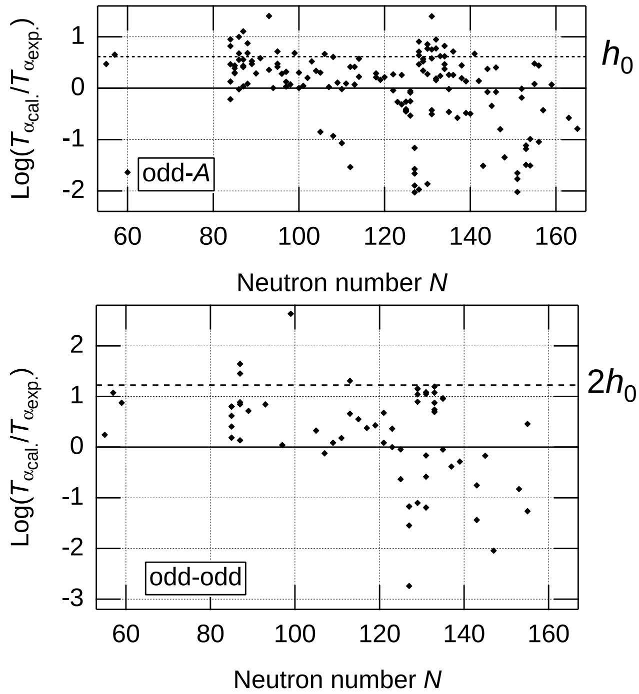


**Figure 2.** Differences between experimental and estimated $T_\alpha$ for odd-$A$ nuclei (upper) and for odd-odd nuclei (lower). The even-odd hindrance factors $h_0$ and $2h_0$ are also seen as dashed lines.

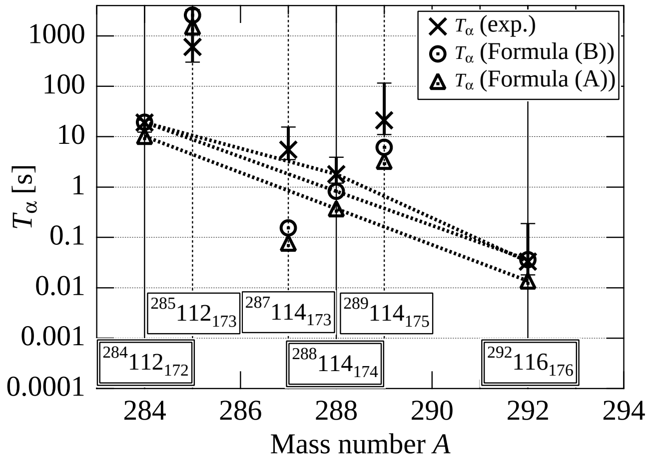


**Figure 3.** Estimated and experimental α-decay half-lives $T_\alpha$ in the superheavy nuclidic region. Dotted lines connect α-decay chains.

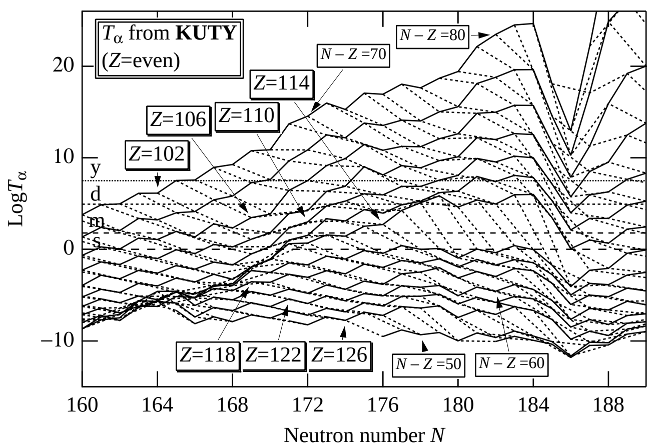


**Figure 4.** $T_\alpha$ of superheavy nuclei by KUTY formula[2,3] for even $Z$. We use Formula (B) to estimate $T_\alpha$. The solid lines connect isotopes and dotted lines connect α-decay chains.

estimated $T_\alpha$ for the superheavy nuclei in Figure 3. In this estimation, experimental $Q_\alpha$ are taken from Reference 1. These nuclei are not input data for parametrization because these were lacking or estimated data in the ENSDF file. This figure shows

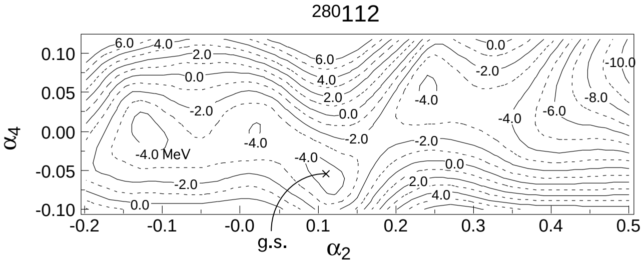


**Figure 5.** Calculated energy surface of $^{280}$112. The ground-state shape of this nucleus is at about $\alpha_2 = 0.11$ and $\alpha_4 = -0.06$.

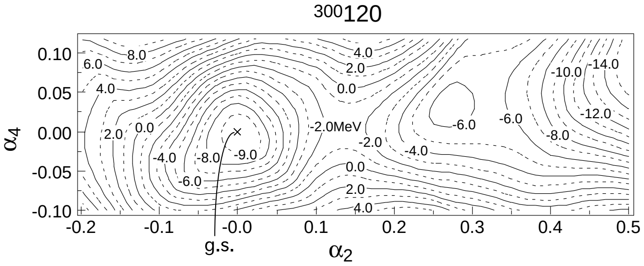


**Figure 6.** Calculated energy surface of $^{300}$120. The ground-state shape of this nucleus is at about $\alpha_2 = \alpha_4 = 0.0$.

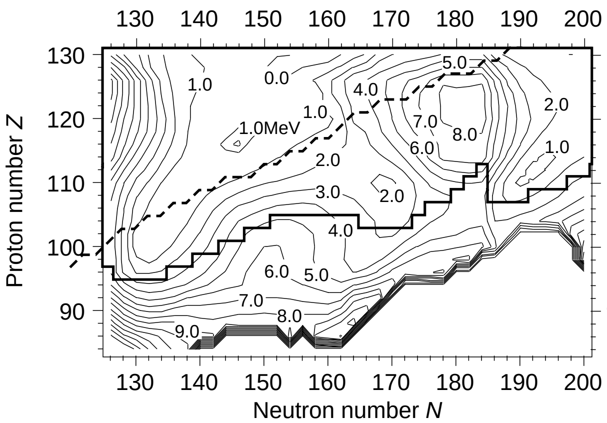


**Figure 7.** Fission-barrier heights for even-even nuclei. The dashed line is the proton-drip line of KUTY formula (even-$Z$). The neutron-rich nuclei located below the solid line may have the higher saddle point in the region $\alpha_2 > 0.5$.

that the estimated $T_\alpha$ are smaller than the measured ones. The values from Formula (B) are relatively larger than those from Formula (A) for the nuclei with large mass numbers.

With the use of Formula (B), we systematically calculate the $T_\alpha$ for superheavy nuclei. In order to estimate the $Q_\alpha$ of superheavy nuclei, we use the KUTY mass formula.[2,3] The result is shown in Figure 4. In this figure our α-decay half-lives present a feature of magicity at $Z = 114$ and at $Z = 126$ as relatively wide gaps between isotope lines, while a similar figure with the use of FRDM mass formula[10] has a larger gap only at $Z = 114$, and that with the use of ETFSI mass formula[11] shows no gap. (The results of FRDM and ETFSI are not shown in the figures.) The magicity at $N = 184$ is also seen as steep decreasing of isotope lines just beyond $N = 184$. The oscillations of the isotope lines are seen because of the even-odd hindrance effect.

## 3. Spontaneous Fission

Although our mass formula is constructed by considering only the equilibrium nuclear shapes, the potential energy surface for spontaneous fission can be calculated by the same method as used for obtaining the shell energies. The fission barrier heights are defined as the highest saddle points from the ground-state shell energies towards the prolate shapes. In this report we take the $\alpha_2$, $\alpha_4$, $\alpha_6$ deformations in the range $-0.2 < \alpha_2 < 0.5$.

We show the energy surfaces against the nuclear deformation for two superheavy nuclei in Figures 5 and 6. For the nucleus $^{280}$112, the height of the fission barrier is only about 2 MeV and its width is relatively narrow. The spontaneous-fission half-life is consequently expected to be rather short for this nucleus. On the contrary, for the nucleus $^{300}$120, the fission barrier height is about 8 MeV, and this width is fairly wide. Therefore, the spontaneous fission of this nucleus is expected to have a very long partial half-life, much longer than the α-decay half-life.

We show the fission barrier heights in Figure 7 for even-even nuclei in the range $84 \leq Z \leq 130$ and $126 \leq N \leq 200$. The nuclei which locate below the solid line may have a higher saddle point in the region $\alpha_2 > 0.5$ because we limited the range on the present calculation.

This figure shows the "hill" of the barrier heights of the nuclei near $^{304}$122. These barrier heights are about 8 MeV or more. Therefore these spontaneous-fission half-lives are expected to be very long. On the contrary, the "basin" of the barrier heights of the nuclei near $^{278}$110 is also seen. These heights are about 2 MeV. There are also other neutron-deficient nuclei having relatively small fission barrier heights whose spontaneous-fission half-lives expected to be rather short.

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